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Effects of Satellite Bunching on the Probability of Collision in Geosynchronous Orbit

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The rapid increase in the satellite population in geostationary Earth orbit is a matter of international concern, in part because of increased collision hazard. Collocated satellite pairs in GEO experience natural drift requiring periodic station-keeping impulses, leading to similar trajectories and close encounters. To assess this risk, a procedure was devised that ranks satellite pairs in GEO according to the highest number of encounters over an extended time interval. Probability of collision was determined by a geometric and a statistical approach. It was found that many pairs of satellites in GEO remain in close proximity and experience many close approaches over time. The top 10 pairs in terms of closest encounters were identified, and mean-time-to-collision based on encounter statistics was determined. Results of the study suggest that the bunching of active or inactive satellites at certain longitudes is a significant effect to be considered in the assessment of the collision hazard in the geosynchronous ring.

Introduction

THE geostationary Earth orbit (GEO) is widely used for communication, broadcasting, weather observation, and surveillance. Physical congestion is an international concern in terms of radio frequency and position allotment, as well as in terms of collision hazard and environmental conservation.[1] The collision hazard is considered as a future issue, and there is neither uniform understanding nor a policy to control it at this time. To reduce the chances of an accidental collision and free desired longitudinal positions, some organizations, including agencies of the U.S. Government, have moved satellites to higher orbits beyond GEO just prior to retiring a satellite from service. Just how effective this practice is and what the altitude of the disposal orbit should be is currently under study. The urgency of the issue is, however, considerable, in view of the fact that the population of GEO spacecraft is increasing at a rate of at least twice that of the general population and may escalate even more in the future.[2,3]

Another important issue is that of collocated satellites in GEO. Active geosynchronous satellites remain fixed over a point on the Earth's surface. Because of the always-present natural perturbations, however, these satellites tend to drift away from their initial Earth-fixed longitudinal positions in time. When two or more of the GEO satellites are placed in or near a specified longitudinal slot, they must be held within a given longitudinal band or "window" by periodic station-keeping impulses. As independent orbit control by station-keeping impulses leads to similar trajectories, the probability of collision at close encounter may become significant. The placement of four satellites at a common longitude, for example, results in an expected time of close encounter (50 m or less) of 0.6 yr assuming uncoordinated station-keeping strategy.[4] The coordinated station-keeping of four satellites located in the 18.8 deg W to 19.2 deg W longitude arc[5], as illustrated in Fig. 1, precludes the possibility of such collision.

A procedure is described that identifies satellite pairs that experience large numbers of close approaches. The personal computer-based process makes use of the USSPACECOM catalog of geosynchronous satellites to calculate the longitudinal position of all satellites at a common epoch. Pairs located within a specified longitude band of 0.5 deg and an orbit inclination band of 1 deg are identified. The orbits of the collocated satellites are then propagated, and the number of close approaches within a given range determined. A ranking of satellite pairs according to the number of encounters is obtained over a time interval of approximately 16 months beginning Nov. 16, 1990. The probability of collision is assessed for the 10 pairs with the highest approach frequencies by a geometrical and a statistical approach.

Study Approach

A typical longitudinal distribution of the geosynchronous population of objects is illustrated in Fig. 2. It is apparent that a significant "bunching" of objects occurs at certain longitudes. The nonuniform distribution of the population is the result of placing active spacecraft at preferred locations for the purpose of improving communication or Earth coverage performance. The maintenance of tight longitudinal boundaries (e.g., < 0.1 deg) is accomplished by a variety of station-keeping strategies, which sometimes ignore the presence of other objects in overlapping regions. In the cases where no coordinated station-keeping is performed, a distinct probability of collision exists.

This study examines closely located or bunched satellite pairs that exhibit large numbers of close approaches over a period of time. The following steps describe the methodology used to perform the study.

1) Using a monthly USSPACECOM database of GEO satellites, propagate all objects to a common epoch.
2) Compute the longitude of all objects at the common epoch.
3) Identify all satellite pairs within 0.5 deg delta longitude and 1 deg delta orbital inclination.
4) Propagate all satellite pairs over a specified time interval, such as two weeks before and after a given date.
5) Rank satellite pairs according to the number of close approaches on a biweekly basis.
6) Select absolute minimum distance as a random variable on a biweekly basis.
7) Compute maximum collision probability by a geometric approach based on the total number of encounters and the position uncertainty of the satellite pairs at encounter.
8) Fit a Weibull distribution function to the absolute minimum values and compute probability of collision.
9) Summarize and compare results.

Encounter Statistics

Typical results obtained by the study approach already described are illustrated in Table 1. The results show that the pair of satellites 15237/16482 experienced the largest number of close approaches within 10 as well as 100 n.mi. range over a period of time between Nov. 16, 1990 to March 16, 1992. The smallest (absolute minimum)

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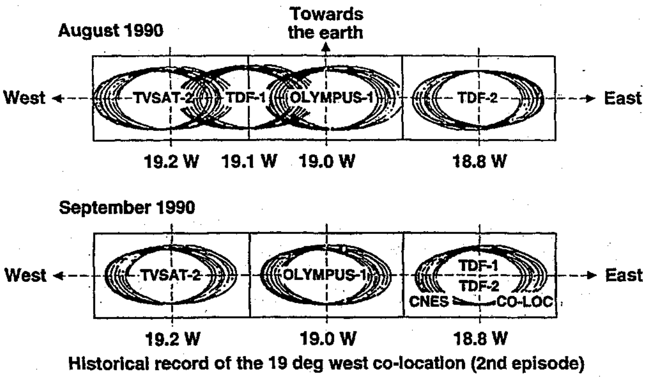

Fig. 1 Example of collocated satellite pairs.

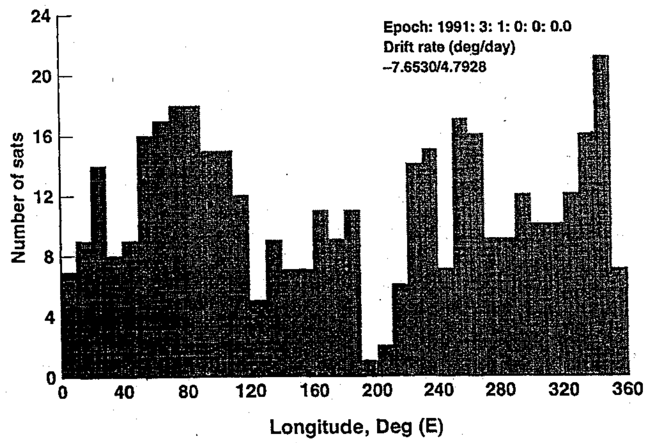

Fig. 2 Geosynchronous satellite longitude distribution.

distance, observed on March 1, 1992, was 0.296 n.mi. based on a biweekly propagation of each satellite. The simulation of close encounters was performed using a compatible NORAD propagator for each of 32 biweekly intervals. No station-keeping maneuvers were simulated, but their effects were assumed to be included in the USSPACECOM two-line element sets supplied on the first of each month for the GEO satellite population.

The sample satellite owner report given in Table 2 gives the longitudinal location and interval between the satellite pairs, as well as the description and launch date information. Figures 3 and 4 illustrate the time history of close approaches for the 15237/16482 satellite pair and the global (absolute) minimum distance (arrows) for each two-week period plotted as a function of time, respectively. This is typical for all pairs of satellites examined.

Table 1 Collocated satellite encounter history, from Nov. 16, 1990 to March 16, 1992

Satellite no. 1	Satellite no. 2	Count of min <10 n.mi.	Count of min <100 n.mi.	Date	ABS min, nm
15237	16482	287	735	03/01/92	0.29600
19621	20705	177	535	12/01/90	0.71400
13069	20872	160	433	11/01/91	0.11600
4902	5587	124	160	10/17/91	0.42900
15826	20946	123	521	12/17/91	0.44100
15235	20873	122	419	08/17/91	0.44400
9047	12309	104	488	05/01/91	0.33400
9852	10365	103	129	05/17/91	0.32100
20193	20762	81	268	06/16/91	1.37400
18384	19344	65	671	11/16/90	0.15300
18316	19874	64	280	03/01/92	2.10000
16101	20776	53	162	01/01/91	6.05700
19330	19684	48	48	02/15/91	0.43800
20107	20217	44	289	04/01/91	0.85400
19397	20693	30	193	07/01/91	0.36000
15642	21222	29	48	08/17/91	1.02900
19621	20122	26	406	08/17/91	0.71900
12967	13984	25	109	02/15/91	0.66000
8697	15643	22	185	11/16/90	4.10300
16597	20771	20	254	04/01/91	3.11500
19548	20777	19	35	01/17/92	0.23200

Table 2 Sample satellite owner report

Satellite ID	Longitude deg, E	Delta longitude deg	Description	Owner	Launch date
19621	340.98	0.0159	TDF 1	FR	10/28/88
20122	340.96	0.0159	OLYMPUS	ESA	07/12/89
15237	274.98	0.0185	TELSTAR 3C	US	08/30/84
16482	274.96	0.0185	SATCOM KU-1	US	01/12/86
19548	297.57	0.0291	TDRS C	US	09/29/88
20777	297.60	0.0291	EUTELSAT II F1	EU	08/30/90
19621	340.98	0.0883	TDF 1	FR	10/28/88
20705	341.06	0.0883	TDF 2	FR	07/24/90
18384	346.20	0.0922	COSMOS 1888	USSR	10/01/87
19344	346.29	0.0922	COSMOS 1961	USSR	08/01/88
9478	182.26	0.1196	MARISAT 3	US	10/14/76
10669	182.38	0.1196	OPS 6391	US	02/09/78
15642	250.73	0.1291	ANIK C1	CA	04/12/85
17561	250.60	0.1291	GOES 7	US	02/26/87
20122	340.96	0.1331	OLYMPUS	ESA	07/12/89
20168	340.83	0.1331	TV SAT 2	FRG	08/08/89
14133	250.93	0.1935	ANIK C2 (TELESAT-7)	CA	06/18/83
15642	250.73	0.1935	ANIK C1	CA	04/12/85
15826	234.98	0.2026	TELESTAR 3D	US	06/17/85
20946	235.18	0.2026	GSTAR IV	US	11/20/90
20107	140.22	0.2102	GORIZONT 18	USSR	07/05/89
20217	140.01	0.2102	GMS 4	JPN	09/05/89
20083	48.60	0.2479	RADUGA 1-1	USSR	06/21/89
21038	48.85	0.2479	RADUGA 1-2	USSR	12/27/90
14077	341.51	0.4434	INTELSAT 5 F-6	ITSO	05/19/83
20705	341.06	0.4434	TDF 2	FR	07/24/90
20193	328.38	0.4436	BSB R-1	UK	08/27/89
20762	328.83	0.4436	BSB R-2	UK	08/18/90

Collision Probability

The probability of two satellites colliding at the time of closest approach is computed by a *geometric* and a *statistical* method using the encounter statistics described previously. The geometric approach is used to compute the maximum probability of collision at each encounter, and the results are summed for all encounters for each satellite pair. The statistical method, on the other hand, employs the Weibull probability distribution fit of the minimum distance between satellites considered as the random variable. This approach represents the application of the asymptotic theory of extreme order statistics to the distribution of minimum distances between different satellite pairs. This approach has been used in Ref. 6, for example, to estimate the collision probabilities for satellites in low Earth orbit.

Geometric Approach

The probability that two satellites will collide during an encounter depends on the distance of closest approach $R_{\min}$ and the physical size of the satellites. The latter may be represented by the sum of the satellite equivalent radii. Thus, for example, the collision radius for two satellites of equivalent radii R_{s1} and R_{s2} is

$$R_s = R_{s1} + R_{s2} \tag{1}$$

If it is assumed that the position uncertainties associated with the three dimensions (coordinates) of a nominal miss distance $R_{\min}$ at an encounter are Gaussian (normal) with zero biases and equal variance σ, and are uncorrelated, a probability of collision for $R_s \ll R_{\min}$ is of the form[7]

$$P(\text{col}) = \left(\frac{2}{\pi}\right)\left(\frac{R_s}{\sigma}\right)^2 \exp\left[\frac{-R_{\min}^2}{(2\sigma^2)}\right] \tag{2}$$

This relationship represents a two-dimensional probability of collision in a plane normal to the relative velocity vector. Because of tracking and ephemeris modeling errors there is an uncertainty in each satellite's position at the time of ephemeris update. The in-track, cross-track, and radial component uncertainties can be combined into a common separation between satellites (e.g., 1 σ), which

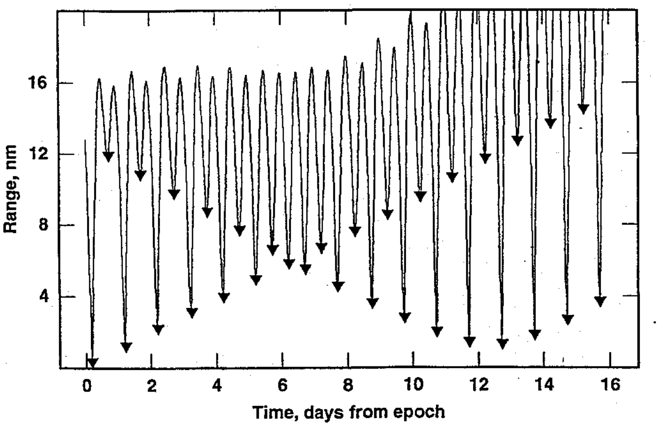

Fig. 3 Range vs time.

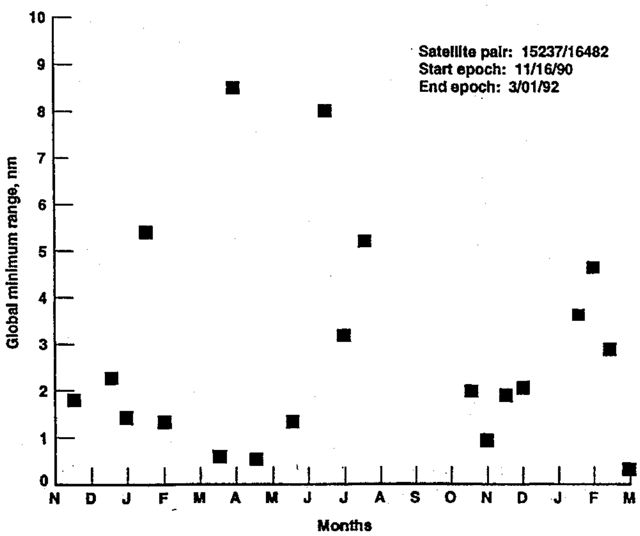

Fig. 4 Global minimum range vs time.

grows as a function of time from the last ephemeris update. Typical uncertainties may be on the order of 0.75 km within two weeks 1.6 km within a month, or 5.0 km within two months. The position uncertainties may, however, be much greater if no recent ephemeris updates exist. This suggests that a worst case uncertainty can be assumed for analysis that yields the maximum probability of collision at the time of closest approach. The result may be regarded as an upper bound, which may be used to determine the relative vulnerability of different satellite pairs.

The maximum value of Eq. 2 occurs when $R_{\min}/\sqrt{2}$ and is given by

$$P(\mathrm{col})_{\max} = \frac{4}{\pi e}\left(\frac{R_s}{R_{\min}}\right)^2 \tag{3}$$

This result is similar to that obtainable as the ratio of the effective cross-sectional area πR_s^2 to the circular area $\pi R_{\min}^2$. Thus, it is a geometric representation of the collision hazard based on the assumption of an equal likelihood that two satellites may be anywhere within a cross-sectional area of radius $R_{\min}$ at the time of the closest approach.

The upper bound probability of collision over an extended time interval (Δ epoch) is the sum of the maximum collision probabilities for all encounters Nt; that is, $\sum_{Nt} P(\mathrm{col})_{\max}$, which is valid when it is less than unity. The corresponding mean time to collision is its reciprocal; that is,

$$Tc = \left(\sum_{Nt} \frac{P(\mathrm{col})_{\max}}{\Delta\,\mathrm{epoch}}\right)^{-1} \tag{4}$$

Tc is summarized in Table 3 where the mean time to collision is seen to be of the order of a few thousand years for the pairs examined. Thus, ranking may be obtained as an indication of the relative collision potential for the satellite pairs of interest.

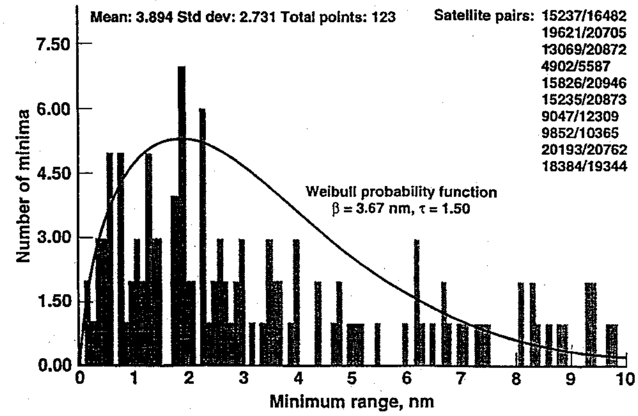

Fig. 5 Global minima for top 10 satellite pairs.

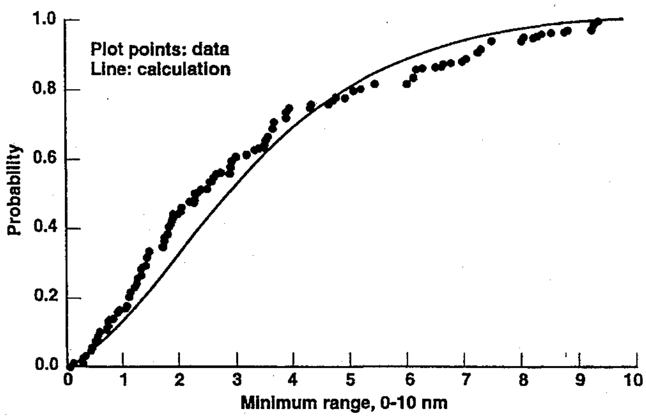

Fig. 6 Weibull probability function $F(x)$, ($\beta = 3.67$ nm, $\tau = 1.50$).

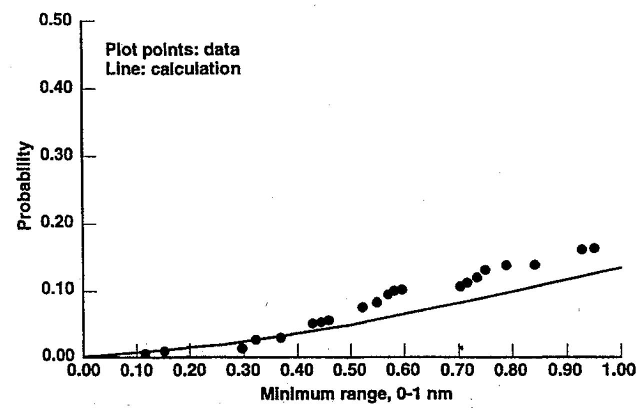

Fig. 7 Weibull probability function $F(x)$, ($\beta = 3.67$ nm, $\tau = 1.50$).

Statistical Approach (Weibull Fit)

The distribution of absolute (global) minimum distances of closest approach obtained for each two-week time interval for all 10 satellite pairs is shown in Fig. 5. A Weibull density function fit to the data in Fig. 5 has been approximated in the form of

$$f(x) = \left(\frac{\tau}{\beta}\right)\left(\frac{x}{\beta}\right)^{\tau-1} \exp\left[-\left(\frac{x}{\beta}\right)^{\tau}\right] \tag{5}$$

where x is the random variable (minimum range) and τ, β are the shape and scale parameters, respectively. The curve fit for $\beta = 3.67$ n.mi. and $\tau = 1.5$ is illustrated in Fig. 5.

It is believed that there is a good correlation with the probability density function of each pair of satellites but it was not possible to

Table 3 Geometric collision probability for all encounters

Pair no.	Satellite pair/ owner	Longitude deg, E	$Nt < 10$ nm in 32—2 weeks per	Rs, M	$\sum_{Nt} P(\text{col})_{\max}/\Delta$ epoch	Tc, yr
1	15237 Telstar (US) 16482 SATCOM (US)	275	287	11	5.16E−05	1614
2	19621 TDF1 (FR) 20705 TDF2(FR)	341	177	19	4.63E−05	1799
3	13069 Westar (US) 20872 SBS6 (US)	261	160	7	4.12E−05	2023
4	4902 NATO2 5587 OPS9431 (US)	252	123	7	7.31E−06	11396
5	15826 Telstar (US) 20946 GSTR (US)	235	122	11	3.19E−05	2609
6	15235 SBS4 (US) 20873 Galaxy (US)	269	124	7	3.06E−05	2727
7	9047 COMSTR2 (US) 12309 COMSTR4(US)	284	103	7	1.44E−05	5772
8	9852 KIKU (JPN) 10365 EKRAN (RUS)	99	104	15	5.89E−05	1415
9	20193 BSB R-1 (UK) 20762 BSB R-2 (UK)	329	81	10	4.59E−05	18165
10	18384 COSI888 (RUS)	346	65	10	4.22E−05	1975
Δ Epoch = 16 months						Average 4950

Table 4 Weibull Collision Probability ($\beta = 3.67$nm, $\tau = 1.5$)

Pair no.	Satellite pair/ owner	Longitude deg (E)	$Nt < 10$ nm, in 32—2 weeks per	Rs, M	Pw(col)/enc $F(x) = (Rs/\beta)^{\tau}$	Pw(col)/month $NtF(x)/16$	Tc, yr
1	15237 Telstar (US) 16482 SATCOM (US)	275	20	11	6.52E−05	8.15E−05	1022
2	19621 TDF1 (FR) 20705 TDF2 (FR)	341	20	19	1.48E−04	1.85E−04	451
3	13069 Westar (US) 20872 SBS6 (US)	261	15	7	3.30E−05	3.10E−05	2691
4	4902 NATO2 5587 OPS9431 (US)	252	6	7	3.30E−05	1.24E−05	6729
5	15826 Telstar (US) 20946 GSTR (US)	235	15	11	6.52E−05	6.11E−05	1363
6	15235 SBS4 (US) 20873 Galaxy (US)	269	10	7	3.30E−05	2.06E−05	4040
7	9047 COMSTR2 (US) 12309 COMSTR4(US)	284	9	7	3.30E−05	1.85E−05	4489
8	9852 KIKU (JPN) 10365 EKRAN (RUS)	99	6	15	1.04E−04	3.88E−05	2145
9	20193 BSB R1 (UK) 20762 BSB R-2 (UK)	329	10	10	5.62E−05	3.52E−05	2370
10	18384 COSI888 (RUS) 19344 COS1961 (RUS)	346	12	10	5.62E−05	4.22E−05	1977
Δ Epoch = 16 months							Average 2778

verify this assumption because of the limited amount of data available for their study. A more appropriate procedure would involve fitting the data points for each pair of satellites with a Weibull or with another function that can best fit the distributions of interest.

The Weibull mode or value of x having the largest associated probability (peak) value is

$$\begin{aligned} m &= \beta[1 - (1/\tau)]^{1/\tau} \\ &= 1.76 \text{ nm} \end{aligned} \tag{6}$$

The mean or expected value is

$$\begin{aligned} \mu &= \beta\Gamma[1 - (1/\tau)] \\ &= 3.31 \text{ nm} \end{aligned} \tag{7}$$

where Γ = gamma function. The standard deviation is given by

$$\begin{aligned} \sigma &= \beta[\Gamma[1 + (2/\tau)] - \Gamma^2[1 + (1/\tau)]]^{1/2} \\ &= 2.25 \text{ nm} \end{aligned} \tag{8}$$

Conclusions

A procedure was described that ranks satellite pairs in geosynchronous orbit according to the highest number of encounters over an extended time interval. The probability of collision was determined by a geometrical and a statistical approach. The geometric method assumed a Gaussian distribution for the position uncertainty of each satellite. The statistical approach used a Weibull probability density function to fit the absolute minimum distance between satellites over a given time interval.

The results of the study show that many pairs of satellites in geosynchronous orbit remain in close proximity to each other and experience large numbers of close approaches over time. The top 10 pairs in terms of the closest encounters were identified as operational satellites that may or may not be subject to coordinated station keeping. The mean-time-to-collision based on the encounter statistics examined was found to range from a few hundred to a few thousand years. These findings are significant in that they identify all satellite pairs undergoing close encounters and present a relative assessment of the collision hazard for such pairs. Moreover, the results suggest that the bunching of active or inactive satellites at certain longitudes is a significant effect that should be considered in the assessment of the collision hazard in the geosynchronous ring. Although the results obtained are approximate in view of the many simplifying assumptions made, they show that the collision hazards for collocated satellites are in general significantly higher than those for the population of objects in general.[8,9]

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